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Computational Models for the Bond between FRP Reinforcing Bars and Concrete

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ABSTRACT: Modeling the behavior of concrete reinforced with fiber-reinforced-polymer (FRP) bars requires constitutive models for concrete and FRP and a model for their interaction. Several models for the constitutive behavior of concrete and FRP have been investigated, but since the combination of these two composite materials to form composite structures is a relatively recent innovation models for their interaction have received less attention. This study focuses on computational models for the mechanical interaction that occurs between FRP bars that have a *significant surface structure* and the concrete matrix they reinforce. This interaction can be computationally modeled at several scales. In a broad sense, the bond models can be grouped into three sets according to the scale of discretization used to idealize the FRP bar. This paper discusses some of the strengths and weaknesses of modeling bond at each of the three scales, reviews the models that have been proposed by researchers at various scales, and reviews some results from recent efforts to characterize the bond behavior at an intermediate scale. This intermediate-scale model is phenomenological in nature, is cast within the mathematical framework of elastoplasticity, and has reproduced limited experimental results with acceptable accuracy.

KEY WORDS: computational model, bond, interface, FRP, reinforcement, bars, concrete, surface structure, mechanical interlocking.

INTRODUCTION

The state of the nation's infrastructure and potential advantages of composite materials has led to increased interest in applying composites to civil engineering structures. This paper focuses on a particular application of composite materials in civil engineering – reinforcing concrete with fiber-reinforced-polymer (FRP) bars. Many researchers have experimentally examined the mechanical behavior of FRP-reinforced concrete structures (see *e.g.*, Refs. [1,2]). Most experimental studies have had a design emphasis, focusing upon structural performance as related to serviceability and safety. There is also a need for computational models of FRP-reinforced-concrete structures or their components. The immediate need is not to provide tools for the civil engineering designer but rather to provide computational models that can help advance the research on these types of structures.

Modeling the behavior of FRP-reinforced concrete requires models for the behavior of concrete and FRP and a model for their interaction. The objectives of the analysis typically define the scale and corresponding idealizations of the model. This study focuses upon modeling the mechanical interaction between FRP bars and concrete. As for any composite material, the interaction between the reinforcement and matrix is important toward understanding the failure of the composite. This interaction (commonly called *bond*) depends significantly upon the *surface* *structure*[1] of the bar. For relatively smooth bars, the bond behavior is affected by misfit conditions from shrinkage and/or swelling, initiation and propagation of interfacial cracks, and friction. For bars with a significant surface structure, the mechanical interlocking of the surface structure with the adjacent concrete further complicates the mechanical interaction. The extent to which the underlying mechanisms are explicitly modeled depends partially upon the scale of the analysis. This paper only addresses models for physical cases where mechanical interlocking is the dominant means of force transfer between the individual materials.

In the specific area of bond between FRP bars and concrete, there have also been many experimental studies (see *e.g.*, Ref. [1], references given by Guo and Cox [3] and the review of Cosenza *et al*. [4]). The tests have primarily focused on determining design criteria, identifying the nature of the bond failure (*e.g.*, failure of the concrete versus failure of the surface structure of the FRP bar), and comparing the bond strength and stiffness of steel and FRP bars.

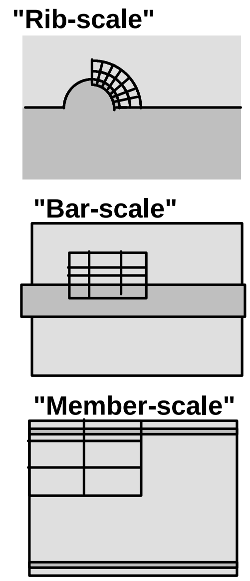


Figure 1. Scales of bond analysis.

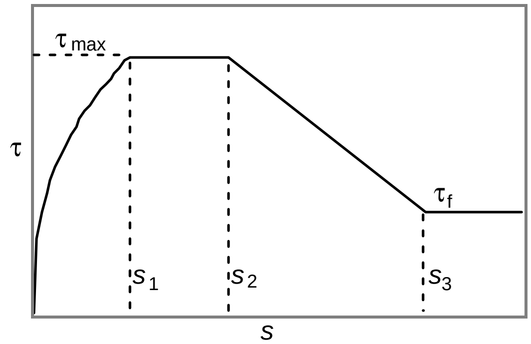
Bond has been numerically modeled at several scales. Figure 1 depicts three scales of bond modeling with names that reflect the scale of spatial discretization [5,6]; however, not all bond models fit neatly into these three categories. The models for bond between FRP bars and concrete have advanced in a manner similar to those for steel bars. Some early experimental data provided the relationship between bond slip and bond stress for specific specimens, and researchers have then defined mathematical models to represent the observed bond stress versus slip behavior. In the context of computations, these models might be referred to as *member-scale* models. The concrete is modeled as a continuum, and reinforcement is modeled using one-dimensional bar elements. At the other "scale extreme" are *rib-scale* models. These models not only represent the reinforcing bar as a solid but also explicitly represent (to differing degrees) the mechanical interlocking of the bar's surface structure with the adjacent concrete. (One might refer to this scale as the "micro-scale" of bond, but this can be confusing since there are also micro-scales associated with both constitutive materials.) The *bar-scale* is a scale of compromise, an intermediate scale in which the bar is modeled as a continuum, but the failure mechanisms associated with the mechanical interlocking are not modeled explicitly. A useful bar-scale model will however be able to reproduce (at a macro-scale) the behavior caused by the various underlying mechanisms for a range of different stress states.

The following sections present overviews of models that have been proposed at each scale and discuss the relative strengths and weaknesses of each type of model. Selected results from recent modeling efforts using a bar-scale model are also presented.

MEMBER-SCALE MODELS

Member-scale bond models are empirical in nature. Generally a few model parameters must be defined to fit the mathematical relationship for bond stress

[1] *Surface structure* refers to the deviation of the actual geometry of the bar from that of a circular cylinder. The surface structure is referred to as being *significant* if it can produce significant mechanical interlocking when relative tangential displacement occurs along the interface.



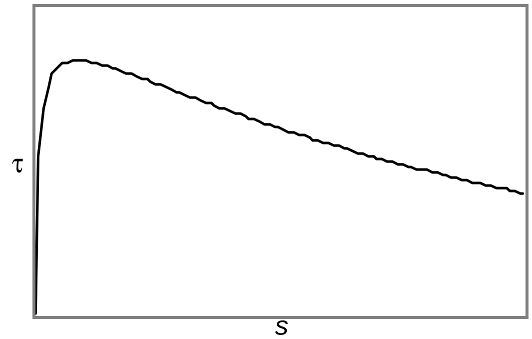


Figure 2. Model of Ciampi *et al*. [6].

Figure 3. Model of Malvar [4,5].

versus slip to experimental data (*i.e.*, calibration is needed). They may also include physical parameters (*e.g.*, a measure of the concrete strength), so that they can be applied to a larger range of conditions without recalibration. Because of their empirical nature many of the models proposed for the bond of steel bars could be applied to the bond of FRP bars even though the underlying mechanisms that affect the bond response may differ significantly. For steel bars, bond models have been proposed for both monotonic and cyclic bond response, but it appears that only bond models for monotonic loading have been applied to FRP bars (thus the scope of this paper).

One of the better known member-scale models for the bond of steel bars was proposed by Ciampi *et al*. [7] and was also adopted in the CEB-FIP model code [8]. This model was based upon the extensive experimental study by Eligehausen *et al*. [9] and is thus often associated with this study. Figure 2 shows a graph of a typical bond stress-slip response for the model. The ascending portion of the bond-slip curve is represented by the relationship

$$\tau = \tau_{\max}\left(s/s_1\right)^{\alpha} \quad \text{for } s \in [0, s_1] \tag{1}$$

where $\alpha<1$, τ ~ bond stress, τ_{max} ~ maximum bond stress, s ~ slip, and s_1 ~ minimum slip at which $\tau = \tau_{max}$. The CEB-FIP model code specifies that α=0.4, and defines τ_{max} and τ_f to be functions of the concrete strength, confinement conditions, and distance from "transverse cracks" (*i.e.*, primary cracks that the reinforcement bridge). The confinement conditions address the potential change in failure mode that can occur with steel bars – shearing of the concrete between the ribs for confined concrete versus splitting of the concrete cover[1] due to the mechanical interlocking. As shown in Figure 2, the remainder of the bond-slip response is idealized as piecewise-linear: an interval of constant response at the maximum bond stress, a linear softening interval, and ending with a constant frictional response.

Faoro [10], Alunno Rossetti *et al*. [11], and Cosenza *et al*. [12] were apparently the first researchers to apply this model to the bond of FRP bars. Cosenza *et al*. [12,13] proposed two slight modifications of the model. First they proposed that the second segment be eliminated (*i.e.*, s_1=s_2). With this modification the response after reaching τ_{max} could be defined in terms of the slope of the softening curve and the magnitude of the frictional response (τ_f). The second modification was a different ascending portion for the bond-slip relationship. They adopted the expression

[1] Consider a cylindrical coordinate system with the z-axis coincident with the axis of the bar. Ideal longitudinal cracks would occur in a θ-coordinate plane. "splitting of the concrete cover" is a failure mode where a longitudinal crack created by the mechanical interlocking propagates from the bar to a free surface that does not intersect the bar.

$$t = t_{\max}\left[1 - \exp\left(-s / s_r\right)\right]^b \tag{2}$$

where s_r and b are curve fitting parameters.

Malvar [14,15] conducted an experimental study examining the effects of confinement stress on the bond response of four different commercially available FRP bars. Malvar proposed a member-scale model to fit his experimental data given by

$$t = t_{\max}\left[\frac{F\hat{s} + (G-1)\hat{s}^2}{1 + (F-2)\hat{s} + G\hat{s}^2}\right] \tag{3}$$

where $\hat{s}$ is the slip divided by the slip at the maximum bond stress (τ_{max}), and F and G are empirical constants that must be determined for each type of bar and different "bond conditions."

Malvar also gave relations to estimate τ_{max} and the slip at this bond stress (s_m) as a function of the confinement stress (σ) and strength (f_t) for his specimen as

$$t_{\max} = A + B\left[1 - \exp\left(-Cs / f_t\right)\right]; \quad s_m = D + Es \tag{4a,b}$$

where A, B, C, D, and E are empirical constant that must be determined for each bar. Figure 3 shows the graph of a typical bond-slip relation given by this model.

For a comparison of how the above models can be calibrated for different experimental sets of data see Cosenza *et al*. [4].

All of the above models share common strengths and weaknesses that are inherent to this scale of idealization. They can allow realistic structural problems to be addressed since the reinforcement can be modeled at a relatively coarse scale using one-dimensional elements. This is important for research that seeks to establish the effects on the overall structural system behavior due to reinforcing structures with different FRP reinforcements. The main weakness of member-scale models is not that they have an empirical basis, but rather that the "scale of empiricism" limits their accurate application to problems which share many common attributes with the calibration specimens. Certainly the effects of some physical parameters can be empirically included in member-scale models (see *e.g.*, Equations 4), but there are limits to these types of improvements. For example, the dependence of the Malvar model upon confinement stress in Equations (4) is valid for his test specimen but is not easily extended to other conditions. Similarly, as suggested in the CEB-FIP code the model of Ciampi *et al*. can be applied under different confinement conditions, but the analysts must specify *a priori* what the conditions are.

For the bond of steel bars, researchers have recently developed bond models that incorporate simplified representations of the longitudinal cracking and of the secondary reinforcement incorporated to constrain this cracking (see e.g., the review of Noghabai et al. [16]). Some of these models were developed to estimate bond strength alone while others were developed to represent the bond stress-slip behavior. The latter models could be applied in member-scale analyses but apparently have not been adopted in the analysis of FRP reinforced structures. One difficult but critical step with these models is relating the bond stress to the radial component of the interface traction (which is primarily responsible for the longitudinal cracking). For steel bars, some researchers have assumed the two traction components are proportional (i.e., that the direction of the interface traction is constant), but this is not consistent with experimental data for either steel or FRP bars [14,15,17]. Including simplified models for the longitudinal cracking is a potential improvement over the member-scale models that have been used for FRP bars; however, (1) the limitations of these models have not been fully established and (2) the level of experimental effort needed to calibrate and validate the models for a variety of structural configurations is significant.

The limitations of the member-scale models highlight the motivation for smaller-scale models – to have a model with a greater predictive capability; unfortunately this comes at a cost since it requires a finer discretization. First we will consider the smallest scale for modeling bond that researchers have examined (rib-scale) and then consider a scale of compromise that has greater predictive ability than member-scale models but can not be used to examine the underlying mechanics of bond failure.

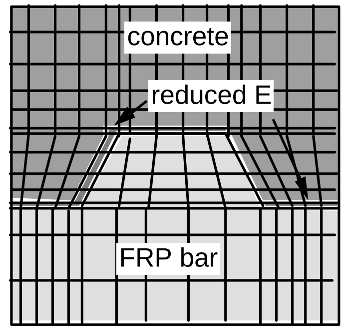


Figure 4. Rib model of Yonezawa *et al*. [18].

RIB-SCALE MODELS

The rib-scale denotes a scale of bond modeling where the surface structure of a bar or its local effects are explicitly modeled (*e.g.*, the complex shape of the bar's surface may be discretized with finite elements). Rib-scale models that do not explicitly model the surface structure of the bar idealize the local effects of the mechanical interlocking. Two idealizations that have been used are (1) limiting the connectivity along the bar-concrete interface to the regions where contact might occur and (2) using nonlinear spring models to characterize the behavior associated with the progressive failures that occur in the contact regions.

In part due to the complexity of the problem, not very many rib-scale analyses have been conducted. Even for steel bars, for which several rib-scale analyses have been conducted, the objective of most analyses has been to provide additional insight to the underlying mechanics rather than to provide quantitative predictions of the bond-slip response. For FRP bars, failure of the mechanical interlocking can involve a progressive failure of the surface structure of the FRP bar as well as local failure of the “interlocking concrete.”

Four rib-scale analyses of FRP bars available in the literature will be reviewed here. All four of these analyses adopt an axisymmetric idealization and use commercial FE systems.

Apparently the first rib-scale analyses for FRP bars were conducted by Yonezawa *et al*. [18]. The objective of their analyses was to optimize the surface structure of GFRP bars for bond. The particular bars of interest were the main bars used in a “three-dimensional reinforcing lattice system.” They performed “two dimensional plane analyses” of bars, examining the effects of surface structure rigidity, spacing, height, and shape upon bond. Their model consisted of ribs having a trapezoidal cross-section (in a θ-plane) evenly spaced along the length of the bar; a schematic of a single rib is shown in Figure 4. Their analyses were linear, thus to account for the concentration of force transfer across the bearing faces of the ribs, they reduced the Young’s modulus in one layer of concrete elements adjacent to the other regions of the interface. They measured the effect of each surface structure parameter (*e.g.*, rib height) by examining the change of key stresses as the parameter was changed. They drew qualitative conclusions about the effects that parameters had on the bond behavior, but the strength of these conclusions must be tempered by the simplicity of the analyses.

Apparently the first rib-scale analyses for FRP bars that addressed the inelastic behavior of the materials were presented by Boothby *et al*. [19]. They were interested in examining how the material properties of the FRP bar affected the bond behavior. They developed axisymmetric FE models of pull-out specimens that had a single rib. Both FRP and steel bars were considered in the analyses. The properties used for the bar were defined to be characteristic of a vinylester matrix reinforced with E-glass fibers (at 50% volume fraction). They modeled the FRP as an anisotropic elastoplastic solid where yielding was defined using a modified Hill criterion. The geometry of the ribs was explicitly modeled, but the use of two-elements to model the rib indicates that a detailed understanding of the failure of the FRP rib was not an objective of the analyses. The concrete was also modeled using an elastoplastic model. The elastic response was assumed to be isotropic linear elastic, but the inelastic component of the model was not described. The interface behavior was modeled using gap-spring elements. A softening law was adopted to model the breakdown in adhesion, and a second spring in series modeled friction. From their analyses they concluded that the transverse compliance and strength of the FRP can change the dominant failure mechanisms in the bond of FRP versus steel bars. In particular they concluded that "the pullout is much more likely to be governed by damage to the reinforcement than by damage to the concrete."

The last rib-scale analyses reviewed here were part of a study conducted to predict the behavior of bars where the matrix suffered environmental degradation. The researchers examined the approach of using the measured response of a bar having only a single rib to help predict how a bar with a longer embedment length would behave. The bars had a vinylester matrix, some reinforced with glass fibers and others with carbon fibers. The ribs had a rectangular cross section and were obtained by machining the surface of a smooth bar. For this specimen the ribs were not very wide and were not integrally tied to the core of the bar by fibers.

A few different rib-scale models were considered. Bakis *et al*. [20] adopted a simplified model of the mechanical interaction without explicitly modeling the geometry of the surface structure. To reduce the computational burden of modeling a larger bond specimen, their model was based upon the experimental observation that the bond failure results from shearing-off the individual ribs from the core of the bar.

They modeled the mechanical interaction by using (1) nonlinear spring elements at the location of each rib to characterize the shear failure of the ribs, and (2) interface elements between the ribs to characterize the breakdown of adhesion and subsequent friction. The load-slip behavior of a single rib was determined empirically by Al-Zahrani et al. [21,22]. He tested single-rib specimens to determine the strength of individuals ribs, and used bar strain measurements for 5-rib and 10-rib specimens to infer the load-slip behavior of a single rib. The test data was represented with sufficient accuracy by a bilinear curve – linear elastic response followed by linear softening. The local bond behavior between ribs was initially estimated from test data for smooth bars, but parametric studies suggested that the contribution of this mechanism to the bond response is not significant compared to the mechanical interlocking of the ribs. The FE model accurately reproduced the experimental data upon which it was based and gave good predictions of: (1) pull-out force versus embedment length, (2) bar-strain versus longitudinal position at different load levels, and (3) load versus bar-strain at different positions.

As previously noted, not all bond models fit neatly into the three defined categories. The model of Bakis *et al*. [20] shared some attributes of bar-scale models. They assume the failure mechanism *a priori* and then empirically determine the load-slip behavior of an individual rib. Unlike bar-scale models the spatial concentration of the mechanical interlocking was explicitly modeled as was adhesion and friction between the ribs. However unlike most rib-scale analyses, their objective was not to examine the underlying failure mechanisms in detail. A potentially important aspect of bond behavior that is not directly addressed in this model is the radial or transverse response. Its effect was indirectly incorporated into the model by empirically determining the load-slip behavior of a single rib, but the radial force created by this interaction was not measured or addressed in the modeling. As a result, one aspect of the study that could not be adequately examined with this model was the effect upon the local bond stress-slip response of bars having internal strain probes. For most of their objectives this simplification was acceptable, but rib-scale models that explicitly model the rib geometry can potentially predict both the longitudinal and radial response.

To examine the local deformation of the ribs in the machined bar in more detail, Uppuluri *et al*. [23] created a thin moiré interferometry specimen from a 1.3 mm slice of the GFRP rod. A plane stress rib-scale model of this specimen was developed which explicitly modeled the rib geometry. Nonlinearity of the model was limited to the interface characterization. The bar and concrete were modeled as orthotropic and isotropic elastic, respectively. Slide lines were used to model the contact conditions between the two materials, but the effects of friction were not included. The model was primarily used to examine the nature of the deformation near the ribs. The FE results were qualitatively consistent with the moiré results, and they found that the transverse stiffness affects the rotation of the ribs which has an affect on the bond strength. This observed behavior helped explain why the previous model (which did not explicitly model the ribs) failed to predict the experimentally observed sensitivity to the internal strain probe.

Uppuluri *et al.* [23] also developed models of bond specimens for two commercially available FRP bars (using axisymmetric approximations for the helical surface structures). The concrete and FRP core were modeled as linear elastic. A resin-rich outer layer of the bar was modeled as elastic perfectly-plastic, and the analysis was halted at the initiation of plastic behavior. The modeling of the outer core was based upon experimental observations of the outer layer peeling away from the core of the bar. The authors indicated that main goal for these models was to "predict the percent variation in the load-slip behavior that occurred with variations in the material properties of the rod." They assumed that the chemical degradation of the elastic moduli of the fibers and matrix were equal (20% to 40%) and that the shear strength of the outer layer was also reduced by a similar amount. The predicted variations in the load magnitude at the onset of nonlinear response were qualitatively consistent with experimental results, but lack of data on the degradation of the constituent properties limited the quantitative comparisons (except in a parametric sense). The authors attributed the inability of the models to predict variations in the slope of the load-slip response with not being able to include interface friction and adhesion due to convergence problems.

The rib-scale models discussed above reflect how modeling at this scale has advanced, but significantly more research is needed to approach the ideal rib-scale model. The ideal rib-scale model could be used to examine the underlying mechanisms of bond and to determine how the properties of the concrete, properties of the FRP, and geometry of the surface structure affect the bond behavior. An accurate analysis capability would provide additional insight on how to optimize bar designs. This ideal is difficult to achieve (in a quantitative sense) for many reasons, among them are: (1) uncertainties in the material properties of the concrete and FRP, and (2) difficulties in accurately characterizing the progressive failure of both materials. For example, concrete is a particulate composite material which is usually modeled as being homogeneous prior to inelastic behavior. This idealization is based upon a representative volume element (RVE) of the material that is certainly valid for examining "bulk material behavior." However, the validity of this type of characterization for rib-scale analyses is questionable, since for these analyses the size of the larger aggregates can be comparable to the size of the region over which high stress gradients occur due to the mechanical interlocking.

While rib-scale models are potentially useful for studying the underlying mechanisms associated with bond, they are currently too computationally demanding to examine the effects of bond at a larger scale (*e.g.*, to examine the behavior of a structural component). Two important failure modes in bond are pull-out and splitting. Rib-scale models can potentially predict both failure modes, but for member-scale models that have been applied to FRP bars the failure mode must essentially be defined *a priori* and then represented by a calibrated function. The need for models that can predict both failure modes under a variety of confinement conditions and that are amenable to larger scale computations motivated the development of a bar-scale model that has been recently applied to FRP bars. The next section presents a brief overview of this model and some recent numerical results.

BAR-SCALE MODELS

Models at the bar-scale are phenomenological in nature, but can potentially predict both pull-out and splitting modes of failure. The model presented here provides a macroscopic characterization of the bond behavior within the mathematical framework of elastoplasticity theory. This model was originally developed for modeling the bond behavior of steel bars [5,6]. The model was more recently applied to the bond of FRP bars. While the failure mechanisms for FRP bars can differ significantly from those of steel bars, the mathematical form of the model was generalized only slightly. The differences in behavior of the two types of bars is principally addressed through the model calibration. For steel bars a single calibration allowed the bond behavior of several different bond specimens to be reproduced with sufficient accuracy, but the surface structure of steel bars is standardized. Several experimental studies have shown that FRP bars with different surface structures can produce significantly different bond-slip behaviors, and the commercial products vary widely. Thus since the initial application of the model to FRP bars did not incorporate physical parameters related to constitutive behavior of FRP, we anticipated that the model would have to be recalibrated for bars having significantly different surface structures. However for the first two studies recalibration was not necessary. The first study addressed the bond of glass FRP (GFRP) bars [3], and the second study addressed the bond of carbon FRP (CFRP) tendons [24].

The model presented has its empirical basis in the experimental work of Malvar [14,15]. Originally these experiments were intended to be one component of a combined experimental-modeling project. The underlying premise of the project was that some of the differences in bond response observed for different specimens could be attributed to the differences in stress states that exist in the specimens. To examine this experimentally, Malvar designed a small cylindrical specimen (75 mm diameter, 100 mm length) that could be tested at different levels of “confinement stress” (Figure 5) A steel band around the specimen was clamped using a hydraulic jack to apply constant confinement stress. The reported confinement stresses were

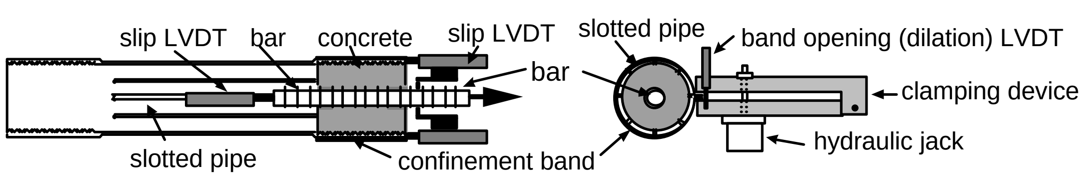


Figure 5. FRP-concrete bond specimen of Malvar [3].

the magnitudes of the average normal tractions at the bar-concrete interface, assuming that the concrete did not carry hoop stress (*i.e.*, when several longitudinal cracks were open). The average radial displacement at the outer surface of the specimen was also obtained by measuring the opening of the steel band. By measuring forces and displacements in both the tangential and radial directions, these experiments provided data for developing a model that coupled the longitudinal and radial responses. Details on the original development and underlying assumptions of the model are presented by Cox and Herrmann [6]. The initial application of the model to FRP bars is presented by Guo and Cox [3]. The following presents a very brief overview of the mathematical form of the model.

The generalized stresses (**Q**) are defined as the homogenized tangent (longitudinal) and normal (radial) components of the interface traction, τ and σ respectively. τ is referred to as the *bond stress*, and $-\sigma$ is referred to as the *confinement stress*. (σ is positive in tension.) The generalized strains (**q**) are defined as the work conjugate tangent (δ_t) and normal (δ_n) displacements of the concrete surface measured relative to the bar surface and nondimensionalized by the bar diameter (D_b); *i.e.*, $\mathbf{q}^T = (\delta_t/D_b, \delta_n/D_b)$. The strains are additively decomposed into elastic ($\mathbf{q}^e$) and plastic ($\mathbf{q}^p$) components. The relationship between stresses (**Q**) and elastic strains ($\mathbf{q}^e$) is assumed to have the linear form: $\mathbf{Q} = \mathbf{D}^e\, \mathbf{q}^e$ where the elastic moduli are defined as

$$\mathbf{D}^e = diag\left[E_c / k_0, E_c /(k_1 + k_2 q_2^p)\right], \tag{5}$$

E_c is Young's modulus of the concrete, and k_0, k_1, and k_2 are model parameters obtained by calibration. A nonzero k_2 introduces elastoplastic coupling which can be used in this context to account for the variation in radial stiffness that occurs with changing contact conditions [25,26].

For monotonic loading, the evolution of the yield surface and flow rule are characterized by a single measure of the internal state, the bond zone "degradation" which is defined as: $d = \min(\frac{\delta_t^p}{s_r}, 1)$ where δ_t^p is the plastic slip and s_r is a characteristic length of the surface structure (*e.g.*, rib spacing).

The yield criterion is of the form $f(\tau, \sigma, d) = 0$ with

$$f(\tau, \sigma, d) = \frac{|\tau|}{f_t} - C(d)\left|\hat{\sigma}(d) - \frac{\sigma}{f_t}\right|^{a_p} \tag{6}$$

where: f_t ~ tensile strength of concrete; C ~ isotropic hardening/softening function; $\hat{\sigma}$ ~ kinematic softening function; a_p ~ a model parameter with a calibration value of 3/4. C and $\hat{\sigma}$ completely characterize the evolution of the yield surface, the dominant feature of which is the softening behavior. C characterizes the effects of the progressive failure of the bond zone, that results in a brief hardening response followed by a softening response; in stress space it opens then closes the yield surface. For some bars $\hat{\sigma}$ can be physically associated with the change in contact forces between the surface structure of the bar and the adjacent concrete; in stress space it translates the yield surface to the origin. The simultaneous occurrence of isotropic hardening and kinematic softening allowed the model to reproduce the observed experimental trend of increasing ductility with increasing confinement stress.

The kinematics of the inelastic wedging action of the ribs is accounted for in the bar-scale model through the flow rule, which initially produces radial dilation of the interface. The following form was adopted

$$\dot{\mathbf{q}}^p = \dot{\lambda}\begin{Bmatrix} \operatorname{sgn}(\tau) \\ g(\sigma, d) \end{Bmatrix} \tag{7}$$

where $\dot{\lambda}$ denotes the consistency parameter, and g is obtained from the analysis of data from the Malvar tests. A key behavior captured by the model is the decrease in radial dilation with an increase in the confinement stress. Unlike member-scale models, the bar-scale model affects the hoop stress near the bar through the radial dilation and thus can produce longitudinal cracking in the adjacent concrete matrix.

Model results have been compared with several bond tests for FRP bars. Each specimen was modeled using axisymmetric FEs. Longitudinal cracking in the concrete was characterized with a cohesive crack model; the bar is modeled as a cylindrical, transversely isotropic solid; and the mechanical interaction is modeled using interface elements that behave according to the bond model. Guo and Cox [3] presented the model calibration and compared the model's prediction of bond strength to fourteen experimental results for GFRP bars (from four independent studies). All but one of the predictions were within 20 percent of the measured values. A calibration result is shown in Figure 6. Note that the bond strength, amount of radial dilation, and overall behavior for this specimen are accurately reproduced by the model. The radial response is equally important as the tangent response; because of the coupled form of this model (i.e., coupling the tangent and radial components), the radial response affects the model's stress state sensitivity and its ability to predict longitudinal cracking.

Cox and Guo [24] recently applied the same model to a particular CFRP tendon (CFCC) for which many test results exist in the literature. The model's predictions were compared with experimental results from four independent studies. Both pull-out and transfer length specimens were considered. For the pull-out specimens all four of the bond strengths were predicted within 20 percent of the measured values.

In prestress applications, the reinforcing tendons are preloaded while the concrete is cast and cured. The prestress load is then relaxed, prestressing the adjacent concrete in compression. A common bond specimen for this application is a long cylindrical specimen with a single tendon. The strains on the outer surface of the concrete are used to determine *transfer length* – the length required to "transfer the bar force into the concrete." Figure 7 shows the predicted and measured concrete strains for a test conducted by Tepfers *et al*. [27]. Based upon the experimental results, Tepfers *et al.* estimated the transfer length to be approximately 400 mm. The specimen had helical secondary reinforcement to reduce the effects of longitudinal cracking, but details on the secondary reinforcement were not given. Two model results are shown in Figure 7. The *unconstrained* case [24] does not account for the secondary reinforcement. The *constrained* case eliminates radial displacement at the outer surface of the specimen, and would thus be representative of the constraint offered by "dense" secondary reinforcement near the outer edge of the specimen. The model results reflect the sensitivity of the model to the stress state, correctly predict a reduction in the transfer length with increased confinement, and agree well with the experimental data.

The use of a single calibration for the two very different types of bars is merely "good fortune," but the ability of the model to reproduce experimental results for a wide variety of specimens indicates that the bond model has a measure of generality. This apparent generality has led to a study where the model in combination with selected experiments is being used to provide useful design data (development length, transfer length, and required concrete cover thickness) for specific CFRP bar – concrete combinations.

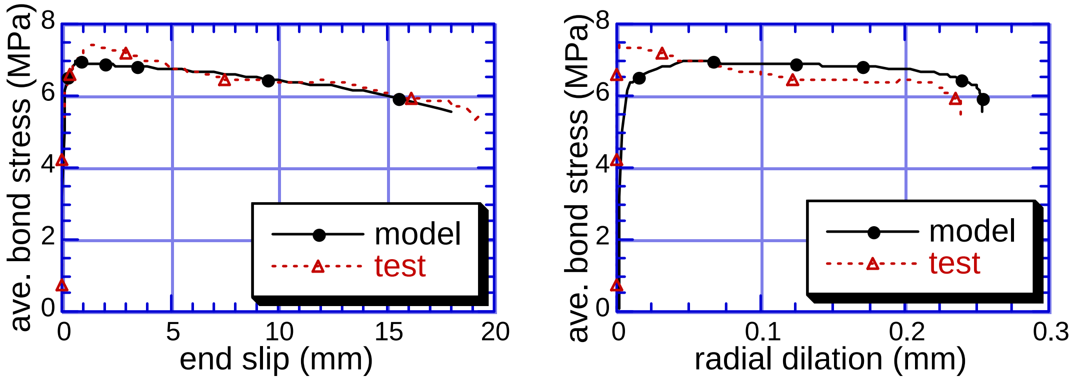


Figure 6. Calibration results for the data of Malvar [4,5] (σ=-1500 psi).

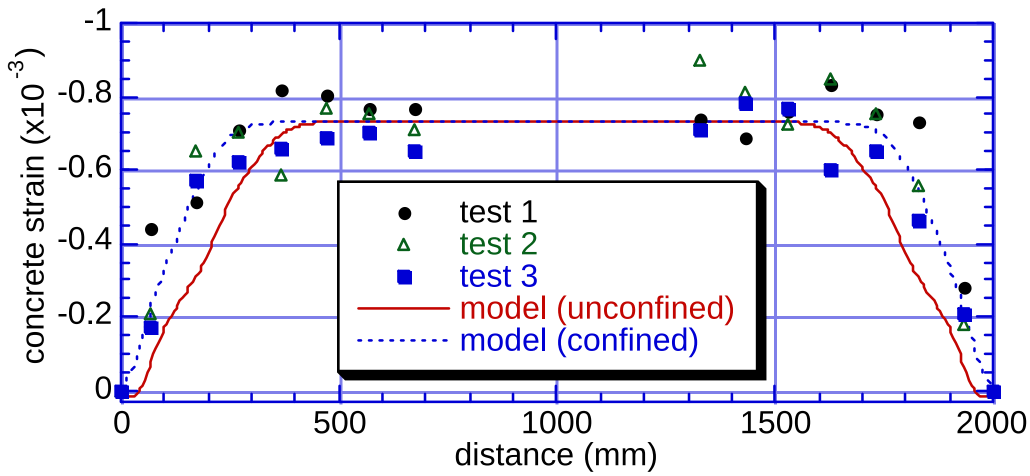


Figure 7. Transfer length tests of Tepfers *et al.* [21]: experiment *vs*. model.

CLOSURE

Currently the primary need for bond models (for FRP bars) is to provide computational tools for advancing research on FRP reinforcing bars, but the potential of using advanced models in performance based design has also been recognized [28]. Most bond models have evolved from models that were previously used for steel bars. The models can be loosely classified according to the smallest scale of discretization used in a FE model. Each scale of modeling addresses the mechanics at a different level of detail, satisfies different technical objectives, and has different strengths and weaknesses that are inherent to the scale. Various member-scale models have been previously proposed that are useful for examining the effects of bond behavior on structural performance. The "scale of empiricism" associated with these models limits their accurate application to problems where the calibration tests accurately represent the state of the "bond zone" (and its evolution) in the actual structure. Rib-scale analyses have the potential to provide additional insight to the progressive bond failure of various FRP bars explicitly accounting for factors such as the surface structure geometry; however, this potential can only be realized with the accurate prediction of the progressive failure of the concrete and FRP surface structure. Bar-scale models provide a characterization of bond behavior at an intermediate scale. They can more accurately account for the stress state than a member-scale model, and their kinematic characterization of the mechanical interlocking can predict a splitting failure. They do not directly characterize the underlying mechanics of bond failure and thus will require recalibration for "significantly different FRP-concrete systems." Further development and application of bond models at the different scales could be used in combination with experimental testing to more effectively advance the application of FRP reinforcement of concrete.

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