# A Dilatational Interface Model for the Bond Behavior of FRP Bars in Concrete

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## Abstract

This study addresses modeling the mechanical interaction (bond) between the surface structure of FRP reinforcements and the adjacent concrete. This complex mechanical interaction can produce damage in both the concrete and the FRP. A phenomenological bond model is presented that uses an interface idealization and incorporates dilation to help characterize the mechanical interaction of the surface structure with the adjacent concrete. The model is formulated within the mathematical framework of elastoplasticity, but it is defined to represent the "macroscopic behavior" associated with the underlying fracture and friction mechanisms that result from the mechanical interaction. In particular, fracture energy associated with local failure of the concrete and/or FRP and the corresponding permanent deformations (both slip and radial dilation) are modeled. The objective of the model is not to provide a detailed description of the underlying mechanics associated with the progressive bond failure, and it will generally require recalibration when applied to significantly different FRP reinforcements. However, this type of model can potentially be used in the detailed analysis of structural components to incorporate the effects of the bond behavior. Selected validation problems using both GFRP and CFRP bars suggest that the model may be sufficiently general (with respect to different stress states) to be applied in larger scale analyses.

## Introduction

Modeling the behavior of FRP-reinforced concrete requires models for the constitutive behavior of concrete and FRP and a model for their interaction (commonly called bond). Reinforcement is designed to prevent and/or bridge cracks that occur in the quasi-brittle concrete matrix. As such, bond behavior is important in determining the nature of localized failures and the amount of energy dissipated by reinforced concrete components. For any composite material, the interaction between the reinforcement and matrix is important toward understanding the failure of the composite. For FRP reinforcements with a significant surface structure, mechanical interlocking dominates the bond response after the initial chemical adhesion is destroyed. There have been numerous tests on the bond behavior of FRP reinforcements, but the analysis of FRP-reinforced concrete has been limited. Bond behavior of reinforcements has been modeled at several scales, each of which has well-defined strengths and weaknesses. So called "*bar-scale* models" are phenomenological models which are often implemented in FE calculations with interface elements. The interface is idealized as cylindrical, the interface tractions are homogenized, and the effects of the underlying mechanics in the bond zone are "lumped" to the interface (see *e.g.*, Cox and Herrmann 1998, Guo and Cox 1999, and Cox and Yu 1999).

This study addresses the characterization of bond behavior between FRP bars and concrete at the bar-scale. A bond model that was originally developed for steel bars is applied to the bond of FRP bars. The model provides a macroscopic characterization of the bond behavior within the mathematical framework of elastoplasticity theory. While the application of the mathematical model is herein extended to FRP bars, the underlying mechanics can be very different, *e.g.*, the effects of the mechanical interaction can produce different failure modes such as the mode II fracture of the FRP bar's surface structure. Since the initial application of the bar-scale model to FRP bars will not incorporate physical parameters related to the constitutive behavior of FRP, we anticipate that the model will have to be recalibrated for reinforcements having significantly different surface structures.

## Elastoplastic Bond Model

The interface model relates the interface traction components to the work conjugate relative displacements. The use of elastoplasticity as a mathematical framework for the model was originally motivated by the classical elastoplastic behavior of many early bond specimens. The model is defined by the generalized stresses and strains, internal variables, yield criterion, elastic moduli, and flow rule.

The generalized stresses ($\mathbf{Q}$) are the tangent ($\tau$) and normal ($\sigma$) components of the interface traction. The generalized strains ($\mathbf{q}$) are defined as the tangent ($\delta_t$) and normal ($\delta_n$) displacements of the concrete surface measured relative to the bar surface and nondimensionalized by the bar diameter ($D_b$); *i.e.*, $\mathbf{q}^T = (\delta_t/D_b, \delta_n/D_b)$. For monotonic loading, the evolutions of the yield surface and flow rules are characterized by a single measure of the internal state defined as $d = \min(\delta_t^p/s_r, 1)$ where $\delta_t^p$ is the plastic slip and $s_r$ is a characteristic length of the surface structure (*e.g.*, rib spacing).

For the application to FRP bars the yield criterion is of the form $f(\tau, \sigma, d) = 0$ with

$$f(\tau,\sigma,d) = \frac{|\tau|}{f_t} - C(d)\left|\hat{\sigma}(d) - \frac{\sigma}{f_t}\right|^{\alpha_p} \tag{1}$$

$f_t$ is the tensile strength of concrete; $C$ is the isotropic hardening/softening function; $\hat{\sigma}$ is the kinematic softening function; $\alpha_p$ is a model parameter with a calibration value of $\alpha_p = 0.75$. The calibration of the model for this study was obtained via simplified analyses of the experimental results of Malvar (1995) for his "type d" GFRP bars. Figure 1 shows how the yield surface model fits the experimental data for a few given states of interface damage. Additional details on the calibration are given in Guo and Cox (1999).

The strains are additively decomposed into elastic ($\mathbf{q}^e$) and plastic ($\mathbf{q}^p$) components. The stress-elastic strain relationship is assumed to have the linear form $\mathbf{Q} = \mathbf{D}^e \mathbf{q}^e$ with the elastic moduli defined as $\mathbf{D}^e = diag(E_c/k_0, E_c/(k_1 + k_2 q_2^p))$, where $E_c$ is Young's modulus of the concrete, and $k_0$, $k_1$, and $k_2$ are model parameters. $k_1$ and $k_2$ define the radial elastic response, which is very important in the prediction of longitudinal cracking. When $k_2$ is nonzero, the elastic response depends upon the plastic dilation (elastoplastic coupling).

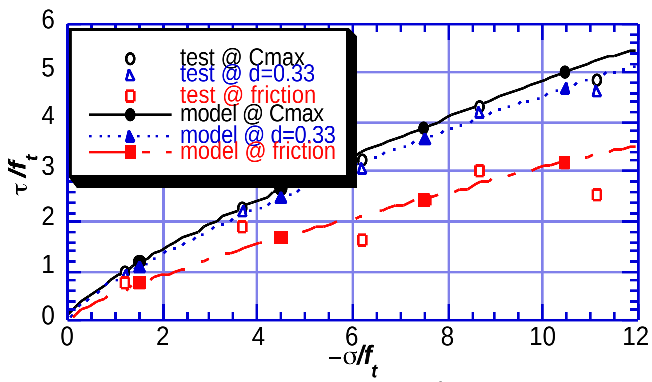

Figure 1. Yield surfaces

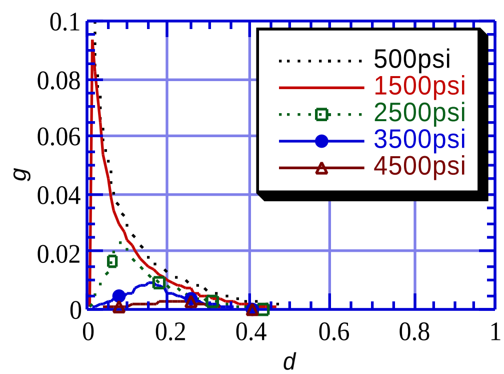

Figure 2. Flow rule

Analytical results that partially support the incorporation of elastoplastic coupling were previously presented (Cox and Yu 1999). The calibrated parameters used in this study are $k_0 = 10$, $k_1 = 0.034$, and $k_2 = 27$.

The kinematics of the wedging action of the surface structure is partially accounted for in the bar-scale model by the flow rule, which initially produces radial dilation of the interface. The following form was adopted for the flow rule description

$$\dot{\mathbf{q}}^p = \dot{\lambda}\begin{Bmatrix} \mathrm{sgn}(\tau) \\ g(\sigma,d) \end{Bmatrix} \tag{2}$$

where $\dot{\lambda}$ denotes the consistency parameter. To obtain an approximation for $g$, limited data presenting radial dilation versus slip (Malvar 1995) were analyzed. Figure 2 shows the calibrated approximation for $g$. The function $g$ quantifies the rate of plastic radial dilation with respect to plastic slip. This data reflects a key behavior that a model of this type must quantify to predict both the bond stress-slip behavior and potential splitting failures: radial dilation decreases with an increase in the confinement stress.

Coupling of the longitudinal and radial responses distinguishes this approach from larger scale bond models that only relate bond stress to slip, because this type of model: (1) actively contributes to the stress state near the bar through the radial dilation and thus can produce longitudinal cracking in the adjacent concrete; and (2) the dependency of the model upon the stress state in the adjacent concrete can potentially provide a more general modeling capability that can be applied to a wider range of conditions without requiring recalibration.

## Validation Results

This section presents validation results for the model. The calibration is based upon the experimental data of Malvar (1995) for a GFRP bar that has a helical indentation (type d). Bond strengths predicted by the model were within ten percent of test values (see Guo and Cox 1999 for additional details). The emphasis of this paper is upon applying the model to predict the bond strength of FRP bars and tendons. To validate the model over a wide range of tests a "particular shape" of GFRP bar and a typical CFRP tendon (CFCC, developed by the Tokyo Rope and Toho Rayon companies) were selected for which several experimental studies are reported in the literature.

All of the bond specimens were modeled using the finite element method (FEM). In all cases, the concrete was modeled as an isotropic elastic material until the hoop stress reached the tensile strength. Longitudinal cracks were incorporated into the models by adopting a "smeared crack" approach in the hoop direction. When unknown, the concrete tensile strength, Young's modulus, and fracture energy were estimated by the empirical relationships given by CEB (1993). The FRP bars and tendons were modeled as transversely isotropic elastic cylinders.

Most of the validation tests considered were pull-out tests. Comparisons of experimental and numerically predicted bond strengths are shown in Table 1. The first three tests on GFRP bars are pull-out tests by Larralde and Silva-Rodriguez (1993), Brown and Bartholomew (1993), and Tepfers *et al*. (1997). Unlike the former two tests in which concrete specimens were cylinders, Tepfers *et al.* specimens were cubic in shape and were cast within strong steel molds. Tepfers *et al.* (1992) adopted similar specimens in CFRP bond tests; in addition, they conducted "ring tests" for CFRP tendons. For these tests the concrete is cast within a steel ring that is strain-gauged to measure the hoop strain. Figure 3 shows a FEM model of this specimen. Figure 4 shows the comparison of experimental data and model predictions. The last pull-out test is based upon data provided by the Tokyo Rope Company (1989) for which two experimental results are given: one for the case of simple monotonic loading, and the second for a monotonic test that occurred after ten load cycles to a tendon force of 34.3 kN. The bond model presented here is only applicable to monotonic loading.

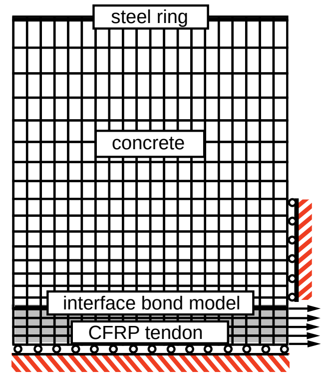


Figure 3. Specimen model of the Tepfer *et al*. ring test.

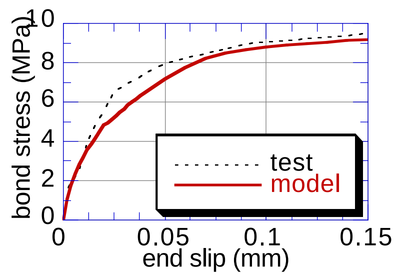


Figure 4. Tepfers *et al.* ring test: exp. *vs*. model.

It can be seen from Table 1 that the dilatational interface model generally predicts the bond strength within an acceptable accuracy. Tepfers *et al.* (1997) concluded for their tests of GFRP bars that "the ultimate bond stress is not influenced by the bond length in a clear way." Their three experimental results shown here (tests 10-12) have bond lengths of $3D_b$, $5D_b$, and $7D_b$, respectively. The difference between average experimental bond strength and that of the model is about nine percent – an acceptable result if the experimental variation is due to scatter.

## Conclusions

Most of the predicted bond strengths were within 15 percent of the measured values. This level of "accuracy" is better than anticipated considering the variation in the specimen properties, the differences in the FRP bars, the uncertainty in the experimental scatter, and the specimen modeling assumptions. Additional studies are needed to quantify some of the experimental uncertainties and modeling assumptions. The application of the

Table 1. Validation Results

| Experiments | | Max. bond stress (MPa) | | |
|---|---|---|---|---|
| | | Test | Model | Difference |
| Larralde *et al.* (1993) (GFRP) | F3-3 | 9.12[1] | 8.97 | 2 % |
| | F3-6 | 8.53[1] | 8.28 | 3 % |
| | F5-3 | 6.35[1] | 5.81 | 9 % |
| | F5-6 | 5.61[2] | 5.51 | 2 % |
| Brown et al. (1993) (GFRP) | Test-A | 8.25 | 9.14 | 11 % |
| | Test-B | 6.84 | 6.05 | 12 % |
| | Test-C | 5.70 | 5.30 | 7 % |
| | Test-D | 3.96 | 4.31 | 9 % |
| Tepfers *et al.* (1997) (GFRP) | No. 10 | 13.9 | 11.8 | 15 % |
| | No. 11 | 9.50 | 11.7 | 23 % |
| | No. 12 | 8.70 | 11.6 | 33 % |
| | Average | 10.7 | 11.7 | 9 % |
| Tepfers *et al.* (1992) (CFRP) | Ring test[3] | 9.46 | 9.20 | 3 % |
| | Pullout[4] | 11.6[2] | 10.6 | 9 % |
| Tokyo Rope (1989) (CFRP) | Mono. load | 7.22 | 6.51 | 10 % |
| | Cyclic load | 6.86 | 6.51 | 5 % |

[1] average of 3 test results
[2] average of 2 test results
[3] bond stress @ slip 0.15mm
[4] bond stress @ slip 0.6mm

phenomenological bond model to FRP bars will generally require recalibration for a particular bar, because the effects of many parameters (*e.g.*, geometry of the surface structure, and volume fraction and orientation of fibers in the surface structure) are difficult to quantify for this type of model. Nonetheless, the preliminary results are encouraging and reflect the potential of using the model to characterize the behavior of concrete reinforced with FRP bars.

## Acknowledgments

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